Flow of Dilatant (Shear-Thickening) Fluids

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The conduit laminar flow of dilatant (shear-thickening) fluids was investigated. It was found that such flow agreed with the Metzner-Reed (friction factor-modified Reynolds number) correlation previously verified only for pseudoplastic fluids. The agreement was found to hold even for cases where the flow behavior index was 2.0 or greater, which caused the velocity term V in the modified Reynolds number $\frac{D^{n'} V^{2-n'}}{g_c 8^{n-1} K'}$ to be raised to the zero or a negative power. It was also demonstrated that conduit laminar flow for dilatant (shear-thickening) fluids could be described solely with the Metzner-Reed correlation and rheological data taken with a small-scale laboratory viscometer. Studies of flow through fittings (90-deg. elbows, globe valves, and couplings) showed a definite effect for non-Newtonian fluids contrary to previous reports for pseudoplastics which indicated essentially Newtonian behavior.

There has been a marked increase in research in the flow of non-Newtonian fluids. One area, however, that has been neglected is the flow of dilatant (shear-thickening) fluids. These fluids are those that show an increase in η_{app}, apparent viscosity

$$\tau = \eta_{app}\, \dot{\gamma} \tag{1}$$

with increasing shear rate. Note that here we mean fluids affected only by shear rate. The term dilatant as used in the present paper does not apply to time-dependent fluids (materials in which apparent viscosity is time dependent). Dilatant (shear-thickening) fluids are materials for which n, the power law exponent

$$\tau = K(\dot{\gamma})^n \tag{2}$$

is greater than 1.

Design procedures for systems involving such fluids can be quite complex. It is essential, therefore, that these procedures be general, rigorous, and yet as simple as possible. Metzner and Reed (*1*) developed for non-Newtonians such a procedure that essentially employed the relationship of a friction factor f to a modified Reynolds number

$$N_{Re}' = \frac{D^{n'} V^{2-n'} \rho}{g_c K' 8^{n'-1}} \tag{3}$$

by the relation

$$f = 16/N_{Re}' \tag{4}$$

The correlation was obtained by using the Mooney-Rabinowitsch (*2*, *3*) relation, together with the expression

$$\frac{D\Delta P}{4L} = K'\left(\frac{8V}{D}\right)^{n'} \tag{5}$$

Metzner and Reed (*1*) then tested the concept that flow behavior could be predicted solely with their correlation and rheological data [n and K derived from shear rate-shear stress curves, not n' and K' derived from Equation (5) above]. They found that the concept worked and that in essence all that was needed for pipe line design for pseudoplastic fluids was small-scale rheological data found with a viscometer and their correlation. They did not however verify that this concept would work for dilatant (shear-thickening) fluids, the main reason being that the combination of flow and rheological data was not available for such fluids.

In considering further the Metzner-Reed correlation certain questions arise. Consider, for example, the case where a dilatant fluid has a n' of 2. In such a situation, the V term drops out of the relation (that is, $V^{2-2} = 1.0$). Furthermore, for n' greater than 2, the V term is raised to a negative power.

The present research was therefore undertaken to determine perimental flow data for dilatant (shear-thickening) fluids; to resolve the questions concerning the applicability of the Metzner-Reed correlation to dilatant (shear-thickening) fluids; and to find if pipe line flow of dilatants could be predicted solely on the basis of rheological (small-scale viscometer) data and the Metzner-Reed correlation.

As noted above, no experimental conduit flow data existed for dilatant (shear-thickening) fluids. There were, however, a number of papers which dealt generally with the rheology of such fluids. These included the work of Reynolds (*4*), Freundlich and Roder (*5*), Williamson and Heckart (*6*), Jobling and Roberts (*7*), Fischer (*8*), Gunnerson and Gallagher (*9*), Metzner and Whitlock (*10*), De Bruijn and Meerman (*11*), Pryce-Jones (*12*), Daniel (*13*), Freundlich and Jones (*14*), Verwey and De Broer (*15*), Griskey and Green (*16*, *17*), and Morgan (*18*). Table 1 summarizes the systems studied by the various investigators.

The foregoing references dealt with the various factors influencing dilatancy, including volume concentration (*4* to *7*, *16* to *18*); particle size, shape, and surface of dispersed solid (*4*, *14*, *16* to *18*); temperature (*16*, *17*); as well as theories of dilatancy (*4* to *7*, *10*, *17*).

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TABLE 2. DILATANT STARCH SYSTEMS STUDIED

Test No.	Specific gravity	Continuous phase	Type of test	K, $(lb._f)(sec.^n)$/sq. ft.	n, dimensionless
75	1.231	Eth. glycol/H_2O	Flow	1.84×10^{-4}	1.60
86	1.222	Eth. glycol/H_2O	Flow	5.22×10^{-4}	1.18
87	1.230	Eth. glycol/H_2O	Flow	10.22×10^{-5}	1.78
88	1.238	Eth. glycol/H_2O	Flow	9.60×10^{-5}	1.79
89	1.238	Eth. glycol/H_2O	Flow	8.72×10^{-5}	1.82
90	1.238	Eth. glycol/H_2O	Flow	5.99×10^{-5}	1.87
93	1.235	Eth. glycol/H_2O	Flow	3.15×10^{-4}	1.45
116	1.226	Eth. glycol/H_2O	Flow	2.85×10^{-6}	2.00
117	1.238	Eth. glycol/H_2O	Flow	4.38×10^{-6}	2.50
98	1.305	Eth. glycol/H_2O/glycerine	Flow	3.08×10^{-3}	1.37
99	1.283	Eth. glycol/H_2O/glycerine	Flow	1.83×10^{-3}	1.35
100	1.260	Eth. glycol/H_2O/glycerine	Flow	1.08×10^{-3}	1.29
114	1.240	Eth. glycol/H_2O/glycerine	Fittings	1.19×10^{-3}	1.15
115	1.243	Eth. glycol/H_2O/glycerine	Fittings	7.51×10^{-4}	1.36

TABLE 1. DILATANT SYSTEMS

Dispersed phase	Continuous phase	References
Gum arabic	Aqueous borax solution	*12*
Corn starch	Aqueous sugar solution	*12*
Glass particles	Water	*4, 11, 12*
Metallic particles	Organic liquids	*8, 15*
Titanium dioxide	Water, sugar solution	*10*
Catalpo clay	Water	*14*
Gypsum, calcium carbonate, quartz	Water	*4, 14*
Quartz, starch	Water	*5*
Vinyl resin	Dioctyl phthalate	*9*
Graphite	Ethylene dibromide, oil	*7*
Metallic oxide pigments	Water	*8, 13, 18*
Starch	Water	*6, 8*
Starch	Carbon tetrachloride	*8*
Calcium carbonate, barium sulfate	Water, formamide, carbon tetrachloride	*8*
Starch	Ethylene glycol	*8*

EXPERIMENTAL PROCEDURE

Flow data were measured in conduits for a variety of dilatant fluids. These fluids were characterized with a cone and plate viscometer (*16, 19*). It was found that all of the fluids followed the power law over the range of shear rates tested (20 to 250 sec.$^{-1}$). Power law parameters are given in Table 2. Typical flow curves are given in Figure 1.

Systems involving starch in water were not used in the present study since they experienced changes in dilatancy with aging. Instead, the systems used were suspension of corn starch in ethylene glycol or ethylene glycol/glycerine with small amounts of water. None of these systems was found to undergo aging (*16, 19*). Specific gravities of the fluids are given in Table 2.

These fluids were prepared by first weighing the correct amount of liquid into a mixing tank, starting a mixer, and then slowly adding corn starch until the required amount was added to the mixture. The mixture was then allowed to agitate for about 2 hr. After this period the mixer was shut off and the mixture was allowed to digest over night. Before an actual flow test in a conduit was initiated, the mixer was started and the starch resuspended and agitated for 2 hr. A typical suspension is shown in Figure 2. All fluids were characterized with the viscometer both before and after an actual flow test in a conduit.

Conduit flow tests were carried out in the apparatus shown in Figure 3. The flow loop included a Moyno pump, a storage tank, agitator, a flow damper, appropriate piping (1¼ in. schedule 40), and manometers.

Care was taken to ensure that the test section was well past the entrance length required for development of the fluid velocity profile. This was done in the following manner: The papers of Bogue (*21*) and Schowalter and Collins (*22*) gave correlations for entrance lengths for Newtonian and pseudoplastic non-Newtonian fluids. These correlations were extended to dilatant (shear-thickening) fluids although not verified with data by the authors (*21, 22*). Entrance lengths were computed for the fluids

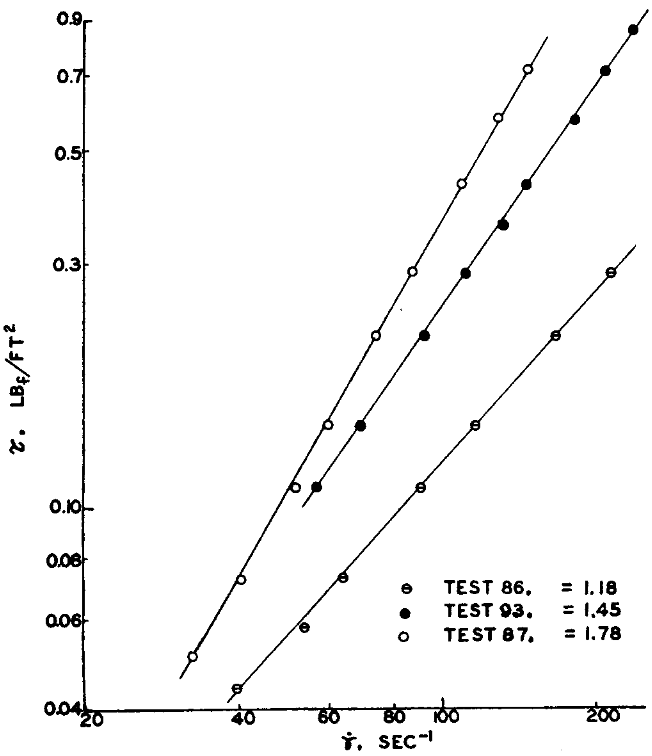

Fig. 1. Typical flow curves for dilatant (shear-thickening) fluids.

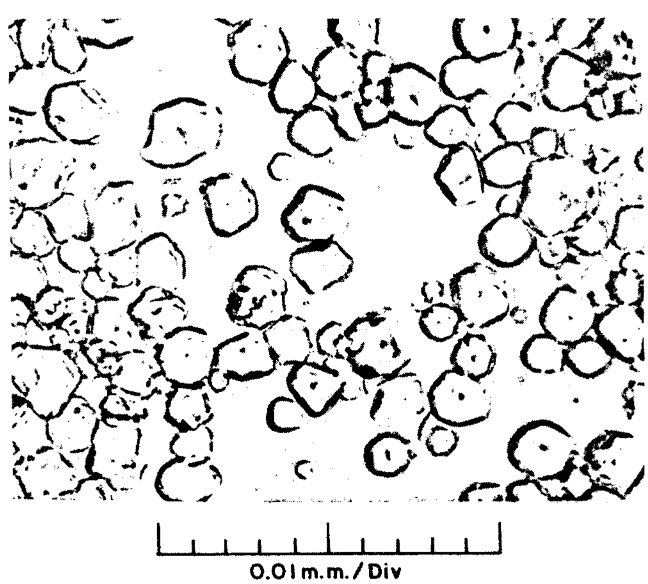

Fig. 2. Photomicrograph of typical suspension.

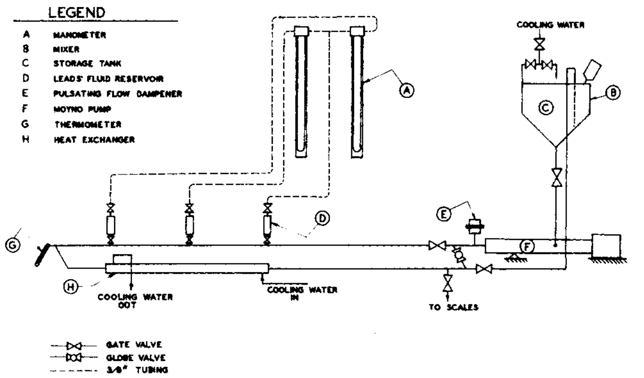

Fig. 3. Flow apparatus.

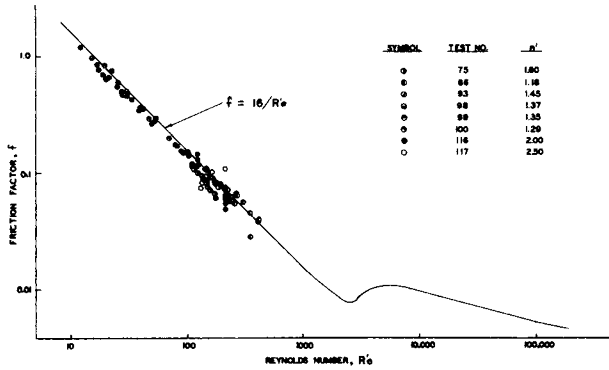

Fig. 4. Experimental dilatant (shear-thickening) fluid data compared to Metzner-Reed friction factor relation.

and flow rates projected in the present study. An entrance length of 70 diameters was then selected for the present study. This length was more than twice the longest calculated entrance length for the most severe conditions of the present work.

The first steps in the experimental procedure were to load the fluid into the storage tank to start the agitator and then to permit the fluid to circulate around the loop. The fluid was then brought to and kept at an equilibrium temperature by means of a heating and cooling system. This system consisted of a 300-w. immersion heater in the storage tank, 8 ft. of double-pipe heat exchangers, and cooling coils in the storage tank.

When the system had come to temperature equilibrium the flow data were recorded. These consisted of pressure drop readings over 5- and 10-ft. pipe lengths, as well as mass rates of flow of the test fluids. The manometers were placed so that they were well beyond the entrance length required for development of velocity profiles in the fluid as noted previously. No noticeable effects of viscous dissipation between manometers were found. A typical set of flow data is given in Table 3.

TABLE 3. REPRESENTATIVE FLOW DATA

Test 117 $n' = 2.50$

Pressure drop P, $lb._f$/sq. ft.	Mass flow rate, lb./sec.	f	N'_{Re}
80.6	1.208	0.0850	133.5
10.6	0.440	0.0842	221.2
29.3	0.744	0.0814	170.2
80.0	1.270	0.0765	129.9
23.2	0.659	0.0826	181.1
87.3	1.250	0.0859	131.4
13.6	0.500	0.1134	207.4
17.8	0.577	0.0824	193.2
20.8	0.647	0.0767	182.6
29.3	0.715	0.0882	173.9
34.1	0.765	0.0897	168.2
56.5	0.910	0.1050	154.0
69.2	0.972	0.1124	149.0
45.6	0.835	0.1004	160.8

DATA AND DISCUSSION

The rheological data of Table 2 were used to compute values of n' and K' experimental values. This was done by the methods suggested by Wilkinson (*20*). These values, together with the appropriate flow data, were then used to calculate values of friction factor and modified Reynolds numbers, as was done previously for pseudoplastics by Metzner and Reed (*1*). The modified Reynolds number values varied from 12 to 400 and average velocities ranged from 0.370 to 1.88 ft./sec.

Figure 4 is a plot of the experimental data compared to the friction factor relation for non-Newtonian fluids as developed by Metzner and Reed (*1*). As can be seen, the dilatant (shear-thickening) fluid data fit the relation rather well. This holds true even where the n' value equals or exceeds 2. All experimental points are within 5% of the predicted values. This precision was considerably better than that obtained for pseudoplastics by Metzner and Reed (*1*). These investigators found average deviations of 10% with some points deviating by as much as 40% in the laminar flow region.

The behavior of fluids with flow behavior index values equal to or greater than 2 is interesting. As was noted at the beginning of this paper there were certain misgivings about the Metzner-Reed correlation, since such fluids would actually have values of modified Reynolds number where velocity V was raised to zero or to a negative power. However, the results of this work show such dilatant (shear-thickening) fluids do definitely agree with the correlation. This means that for cases where n' exceeds 2 the friction factor values actually increase with increasing velocity rather than what normally occurs with Newtonian and pseudoplastic non-Newtonian fluids (decreasing friction factor values with increasing velocity). This apparent anomaly takes place because velocity V is raised to a negative power (that is, $V^{2-n'}$) in such cases. This means that the modified Reynolds number decreases with increasing velocity and therefore friction factor values increase. The entire effect was shown rather forcefully when attempts were made to carry out flow tests on fluids with n' values exceeding 2.5. In these cases pressure drops were too high to be recorded by our manometer system. This meant that friction factor values must have been quite large because of greatly reduced modified Reynolds numbers.

The flow of dilatant (shear-thickening) fluids through fittings (globe valve, 90-deg. elbow, and coupling) was also studied. There were no previous investigations of this type reported in the literature. However, some work had been reported for pseudoplastics (*23*, *24*). The fluids used in the present work are identified in Table 2 as tests 114 and 115.

Figure 5 shows the equivalent lengths as a function of modified Reynolds numbers for 90-deg. elbows. The globe valve data showed the same trends. However, no effect was found for couplings (as would be expected). These data show that the equivalent length apparently decreased with increasing Reynolds number and that there is an effect of n, the flow behavior index, on the equivalent length behavior. Both of these deserve comment. In the latter case the effect of n would seem to go against the previous work on pseudoplastics (*23*, *24*) which generally concluded that there was no effect of non-Newtonian behavior on flow through fittings. It should be noted, first, that the papers on pseudoplastic fluids were sketchy and not too well-defined. In addition, other researchers on entrance effects or entrance lengths (*21*, *22*) have definitely shown an effect of n. This would refute the prior findings on pseudoplastic fluids. Furthermore, the effect of n in Figure 5 agrees with the extrapolation to dilatant fluids ($n > 1.0$) of the relation derived by Showalter and Collins (*22*) for entrance lengths.

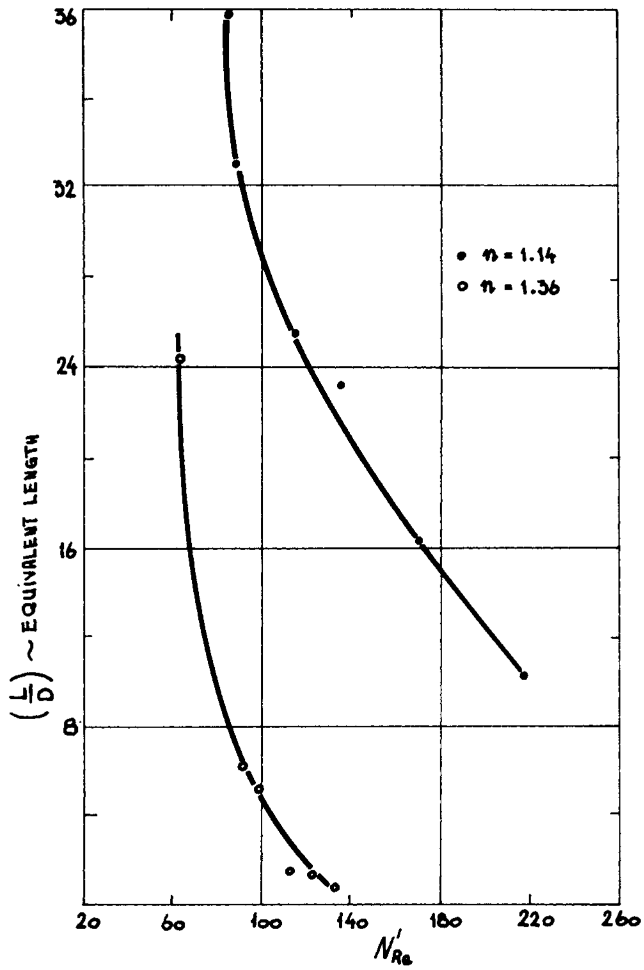

Fig. 5. Equivalent lengths for 90-deg. elbow versus modified Reynolds number.

The decrease of equivalent length with increasing N'_{Re} concurs with the studies of Pigott (*25*) and Freeman (*26*) for Newtonians. These investigators found that the pressure drop due to bend losses decreased with increasing Reynolds number and then leveled off in the turbulent flow region.

This work, and the lack of firm results for pseudoplastics, suggest that a more thorough study of flow of non-Newtonians through fittings should be undertaken.

CONCLUSIONS

1. The Metzner-Reed correlation of friction factor with modified Reynolds number was found to hold for dilatant (shear-thickening) fluids in laminar flow.

2. Conduit laminar flows of dilatants can be predicted solely on the basis of the Metzner-Reed correlation and rheological data taken with a laboratory viscometer.

3. The velocity term $V^{2-n'}$ in the modified Reynolds number did not affect the correlation adversely for $n' \geq 2.0$. for laminar flows of dilatant (shear-thickening) fluids.

4. Non-Newtonian behavior was found to influence the equivalent lengths needed to account for fittings.

ACKNOWLEDGMENT

Grateful acknowledgment is made to the National Science Foundation for support of this research by means of Grant GP-3158.

NOTATION

$f = \dfrac{\Delta P g_c D}{2L\rho V^2}$, dimensionless
g_c = (32.17 $lb._m$)(ft.)/($lb._f$)(sec.^{2})
n = flow behavior index, dimensionless [Equation (2)]
n' = flow behavior index, dimensionless [Equation (5)]
r = radius, ft.
t = time, sec.
v = velocity, ft./sec.
w = subscript indicating wall conditions
D = diameter, ft.
K = consistency index, ($lb._f$)(sec.^{n})/sq. ft. [Equation (2)]
K' = consistency index, ($lb._f$)(sec.^{n})/sq. ft. [Equation (5)]
L = length, ft.
N'_{Re} = modified Reynolds number
ΔP = pressure drop, lb./sq. in.
Q = volumetric flow rate, cu. ft./sec.
V = average velocity, ft./sec.

Greek Letters

γ = strain
ϕ = function
ρ = density, $lb._m$/cu. ft.
τ = shear stress, $lb._f$/sq. ft.
μ = viscosity, ($lb._f$)(sec.)/sq. ft.
η_{app} = apparent viscosity, ($lb._f$)(sec.)/ft.
$\dot{\gamma}$ = shear rate, sec.$^{-1}$

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Manuscript received August 6, 1969; revision received October 29, 1969; paper accepted November 3, 1969. Paper presented at AIChE Salt Lake City meeting.